Chap. 10 Beyond the Macroscopic Quantum Model (Ginzburg-Landau Theory)

Section 10-1 : Introduction

1. A superconductor with a uniform density of electrons and in the absence of any electromagnetic (EM) fields
→Gibbs free energy in the GL theory is written in terms of the superelectron density $\mathbf{n}^*$.

2. Incorporate non-uniform density of electrons and EM fields
→The order parameter $\Psi(r)$ will be used to derive the characteristic length (the coherence length ξ)

Section 10-2 : Ginzburg-Landau Theory

The condensation energy was defined in terms of the difference Gibbs free energies for the normal and superconducting states, Eq. (6-112):

$$G_s(0,T) - G_n(0,T) \equiv -\frac{1}{2}\mu_0 H_c^2(T) V_s \quad \text{-----} (10\text{-}1)$$

Where, $H_c = \dfrac{\Phi_0}{2\sqrt{2}\pi\mu_0(\xi\lambda)}$ is the thermodynamic critical field.

GL Free Eneray for subsystem (A)

$$G_A(0,T,n_s^*) = G_n(0,T) + V_s\left(\alpha(T) n_s^* + \frac{1}{2}\beta(T)\left(n_s^*\right)^2\right)$$

where n_s^* is the superelectron density.

Both $\alpha(T)$ and $\beta(T)$ are coefficients independent of n_s^*.

$$G_A(0,T,n_s^*) = G_n(0,T) \quad \text{for } n_s^* = 0.$$

Figure 10-1. The difference in the Gibbs free energies between the subsystem A and the normal state,

$$G_A(0,T,n_s^*) - G_n(0,T) = V_s\left(\alpha(T)n_s^* + \frac{1}{2}\beta(T)n_s^{*2}\right) \Leftarrow \beta(T) > 0$$

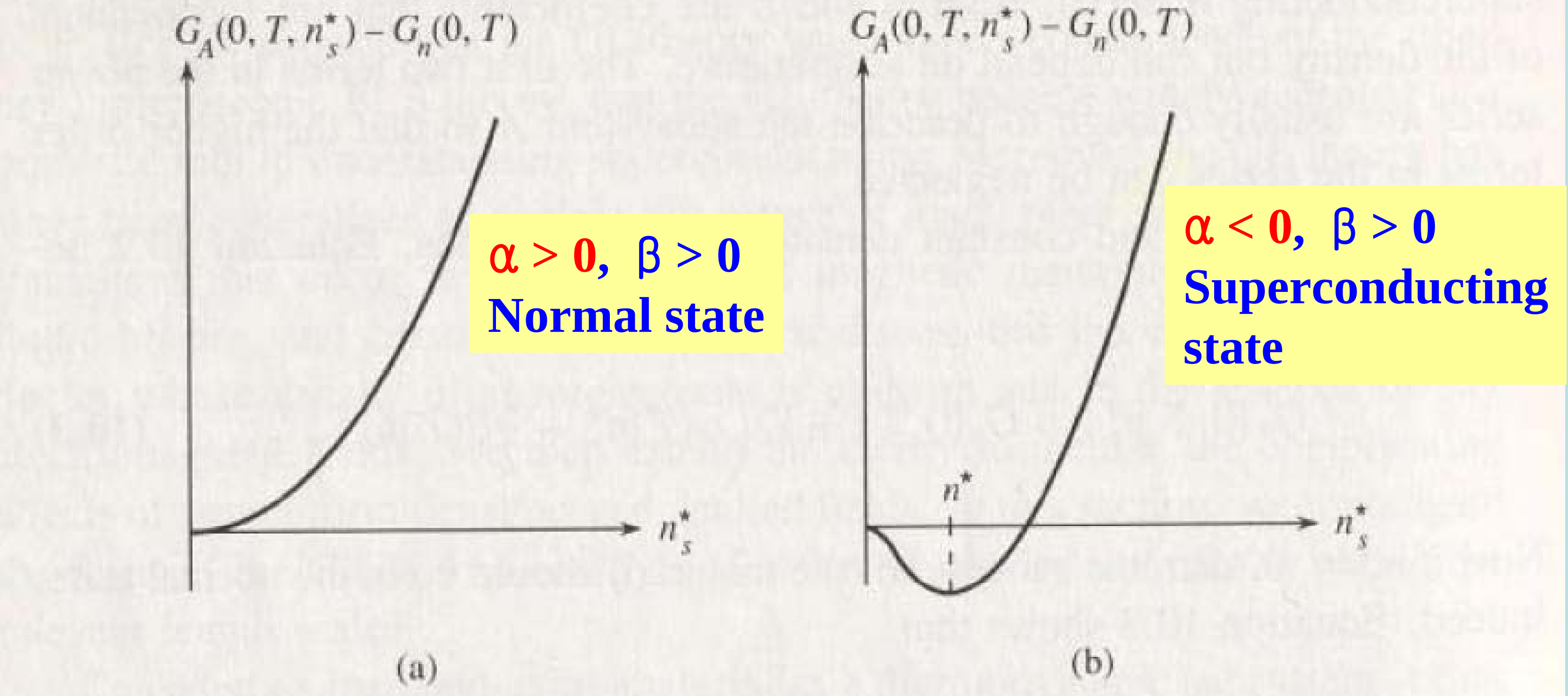


Figure 10.1 The difference in the Gibbs free energies between the subsystem A and the normal state for (a) $\alpha > 0$, which describes the normal state, and (b) $\alpha < 0$, which describes the superconducting state.

T dependence of H_c and n*

$$G_A(0,T,n_s^*) = G_n(0,T) + V_s\left(\alpha(T)n_s^* + \frac{1}{2}\beta(T)n_s^{*2}\right)$$

The equilibrium density can be found;

$$\left(\frac{\partial}{\partial n_s^*} G_A\left(0,T,n_s^*\right)\right)\Bigg|_{n_s^*=n^*} = 0 \Rightarrow n^* = -\frac{\alpha}{\beta}$$

$$n^* = \begin{cases} 0 & \text{for } \alpha > 0 \Leftarrow \text{normal state} \\ -\frac{\alpha}{\beta}V_s & \text{for } \alpha < 0 \Leftarrow \text{superconding state} \end{cases}$$

Free energy in the SC state ($A \Rightarrow S$);

$$G_S(0,T,n^*) = G_n(0,T) - \frac{1}{2}\frac{\alpha^2}{\beta}\mathrm{V_s} \Rightarrow G_S < G_n$$

From, $G_s(0,T) - G_n(0,T) = -\frac{1}{2}\mu_0 H_c^2(T)V_s$

$$-\frac{\alpha^2}{2\beta} = -\frac{1}{2}\mu_0 H_c^2(T)V_s$$

$$\alpha = -\frac{\mu_0 H_c^2}{n^*}, \quad \beta = \frac{\mu_0 H_c^2}{(n^*)^2}$$

$$G_A = G_n - \mu_0 H_c^2 V_s\left(\frac{n_s^*}{n^*} - \left(\frac{n_s^*}{n^*}\right)^2\right) \text{ from (10-1)}$$

where n^* the equilbrium density of superelectron.

(1) T dependence of $H_c(T)$

Experimentally, $H_c(T) = H_{c0}\left(1 - (T/T_c)^2\right)$

$$\lim_{T\to T_c} H_c(T) = H_{c,GL}(0)\left(1 - \frac{T}{T_c}\right) \text{ near } \mathrm{T_c}\ (T/T_c << 1).$$

$H_{c,GL}(0) = 2H_{c0} \Leftarrow$ GL critical field.

"$H_{c,GL}(0)$ is not zero-temperature value!!!"

(2) T dependence of $n^*(T)$

Experimentally, $n^*(T) = n_0^*\left(1 - (T/T_c)^4\right)$

$$\lim_{T\to T_c} n^*(T) = n_{GL}^*(0)\left(1 - \frac{T}{T_c}\right) \text{ near } \mathrm{T_c}\ (T/T_c << 1).$$

$n_{GL}^*(0) = 4n_0^* \Leftarrow$ GL superelectron density.

The order parameter (Ψ) and Coherence length (ξ)

$$G_A(0,T,n_s^*) = G_n(0,T) + V_s\left(\alpha(T)n_s^* + \frac{1}{2}\beta(T)n_s^{*2}\right)$$

GL free energy in the magnetic field where n_s^* depends on position, given by :

$$G_s(\vec{H},T) = G_n(0,T) + \int_{V_S}\left(\alpha n_s^* + \frac{1}{2}\beta n_s^{*2} + \frac{\hbar^2}{2m^*}\left(\nabla\sqrt{n_s^*}\right)^2\right)dv + \frac{1}{2\mu_0}\int_{V_s}(B^2 + \mu_0\Lambda_s J_s^2)dv$$

where $\frac{\hbar^2}{2m^*}\left(\nabla\sqrt{n_s^*}\right)^2$ and $\Lambda_s = \frac{m^*}{n_s^*(q^*)^2}$ are position - dependent functions of superelectron.

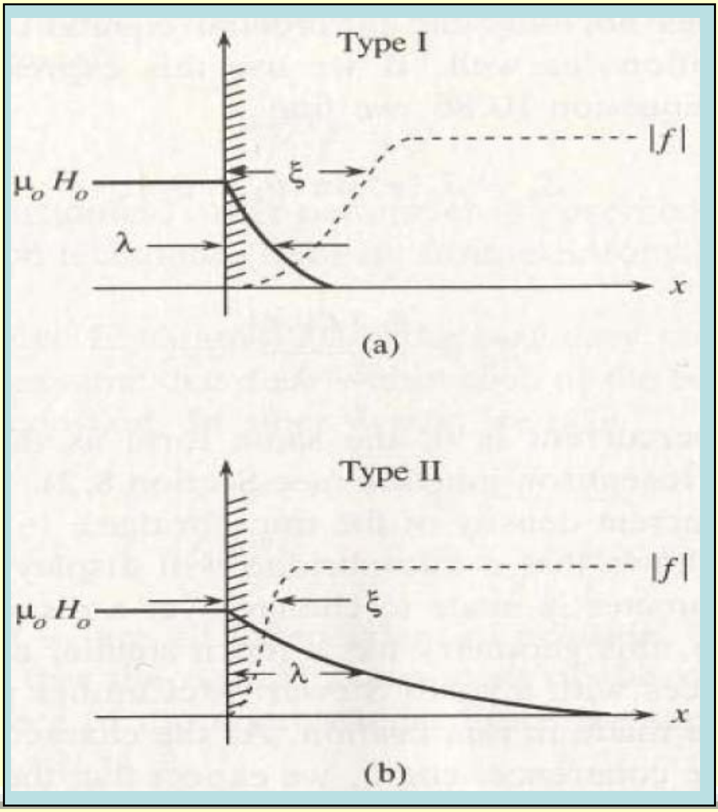
Since $\alpha = -\frac{\mu_0 H_c^2}{n^*}$, $\beta = -\frac{\mu_0 H_c^2}{(n^*)^2}$

$$\Delta G = -\mu_0 H_c^2 \int_{V_S}\left\{\frac{n_s^*}{n^*} - \frac{1}{2}\beta\left(\frac{n_s^*}{n^*}\right)^2 - \xi^2\left(\nabla\sqrt{\frac{n_s^*}{n^*}}\right)^2\right\}dv$$

$$+\frac{1}{2\mu_0}\int_{V_s}(B^2 + \mu_0\Lambda\frac{n_s^*}{n^*}J_s^2)dv$$

[The characteristic length (ξ) is defined as]

$$\xi^2 \equiv -\frac{\hbar^2}{2m^*\alpha} = \frac{\hbar^2 n^*}{2m^*\mu_0 H_c^2} \Leftarrow \Phi_0 = \frac{2\pi\hbar}{|q^*|}$$



T dependence of coherence length (ξ)

$$\xi^2 = \frac{\Phi_0^2}{8\pi^2\mu_0^2\lambda^2 H_c^2} \Rightarrow \xi = \frac{\Phi_0}{2\sqrt{2}\pi\mu_0\lambda(T)H_c(T)}$$

$$\lim_{T\to T_c}\lambda(T) = \frac{\lambda_{GL}(0)}{\sqrt{1-(T/T_c)}}, \quad \lim_{T\to T_c} H_c(T) = H_{c,GL}(0)\left(1-\frac{T}{T_c}\right)$$

$$\lambda(T)H_c(T) = \lambda_{GL}(0)H_{c,GL}(0)\sqrt{1-(T/T_c)}$$

$$\xi = \frac{\Phi_0}{2\sqrt{2}\pi\mu_0\lambda_{GL}(0)H_{c,GL}(0)\sqrt{1-(T/T_c)}} = \frac{\xi_{GL}(0)}{\sqrt{1-(T/T_c)}}$$

$$\Rightarrow \xi = \frac{\xi_{GL}(0)}{\sqrt{1-(T/T_c)}}, \quad \xi_{GL}(0) = \frac{\Phi_0}{2\sqrt{2}\pi\mu_0\lambda_{GL}(0)H_{c,GL}(0)}$$