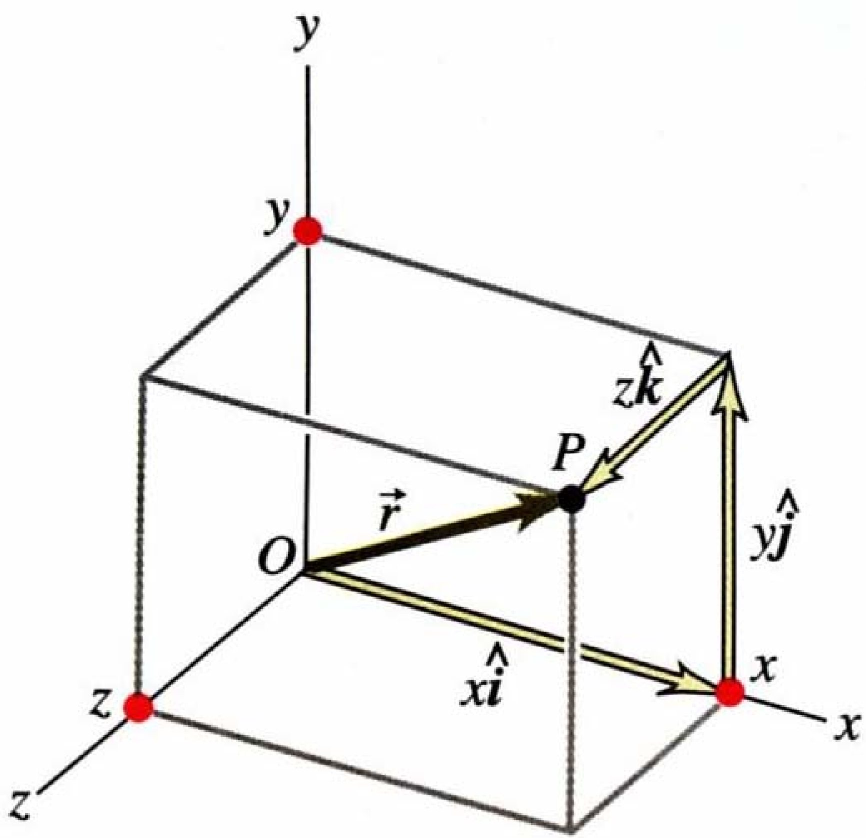
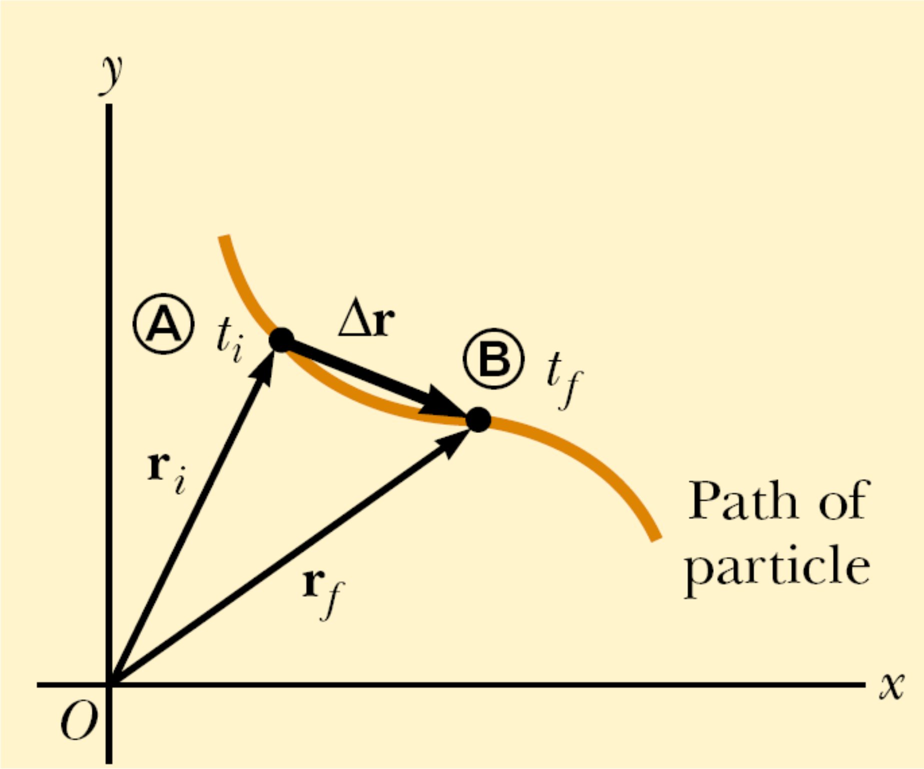
제4장 2 차원 운동

4-1 위치(r), 속도(v), 가속도(a) 벡터





위치벡터 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

변위벡터 $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$\vec{r}_i = x_i\hat{i} + y_i\hat{j}, \quad \vec{r}_f = x_f\hat{i} + y_f\hat{j}$

$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$

• 속도 벡터(**v**)

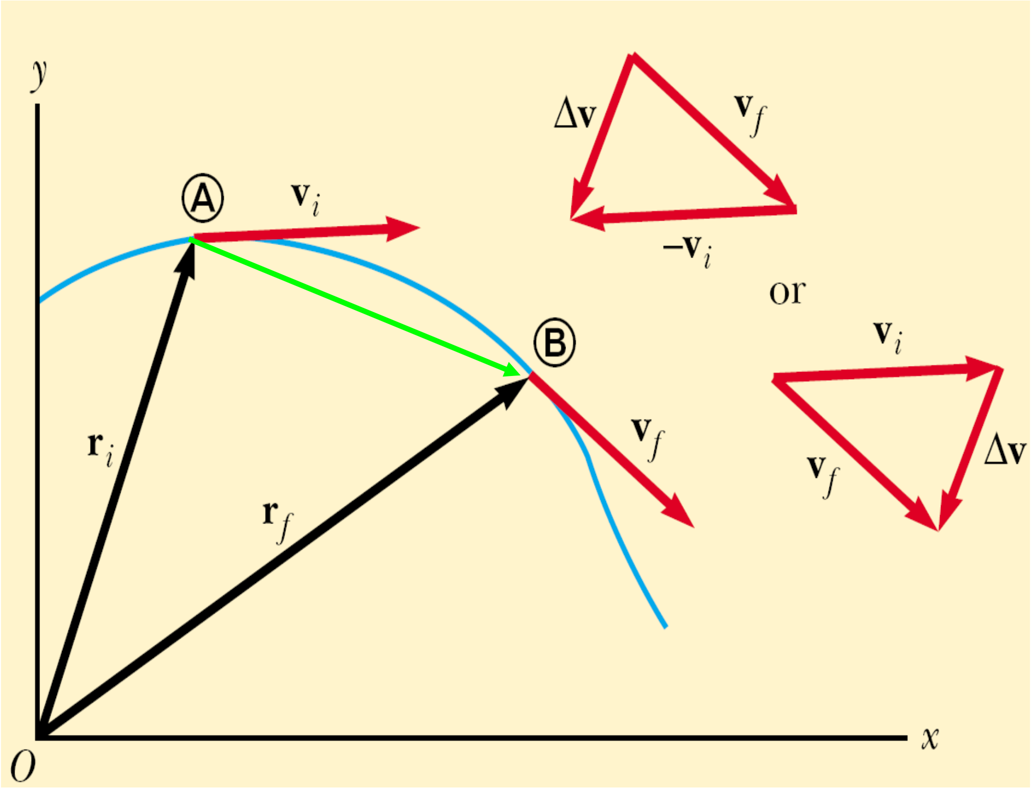
$$\text{속도 } \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$= v_x \hat{i} + v_y \hat{j}$$

$$= \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j}$$

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}$$

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2}$$



• 가속도 벡터(**a**)

$$\text{가속도 } \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}, \quad \Delta \vec{v} = \vec{v}_f - \vec{v}_i$$

$$= a_x \hat{i} + a_y \hat{j} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} = \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j}$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}, \; a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$

$$|\vec{a}| = a = \sqrt{a_x^2 + a_y^2}$$

4-2 등가속도 2차원 운동

$$\vec{r} = x\hat{i} + y\hat{j}, \quad \vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$v = v_0 + at, \quad x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$t = 0; \quad x_i, y_i, v_{xi}, v_{yi} \Rightarrow$$

$$t = t; \quad x_f, y_f, v_{xf}, v_{yf} = ?$$

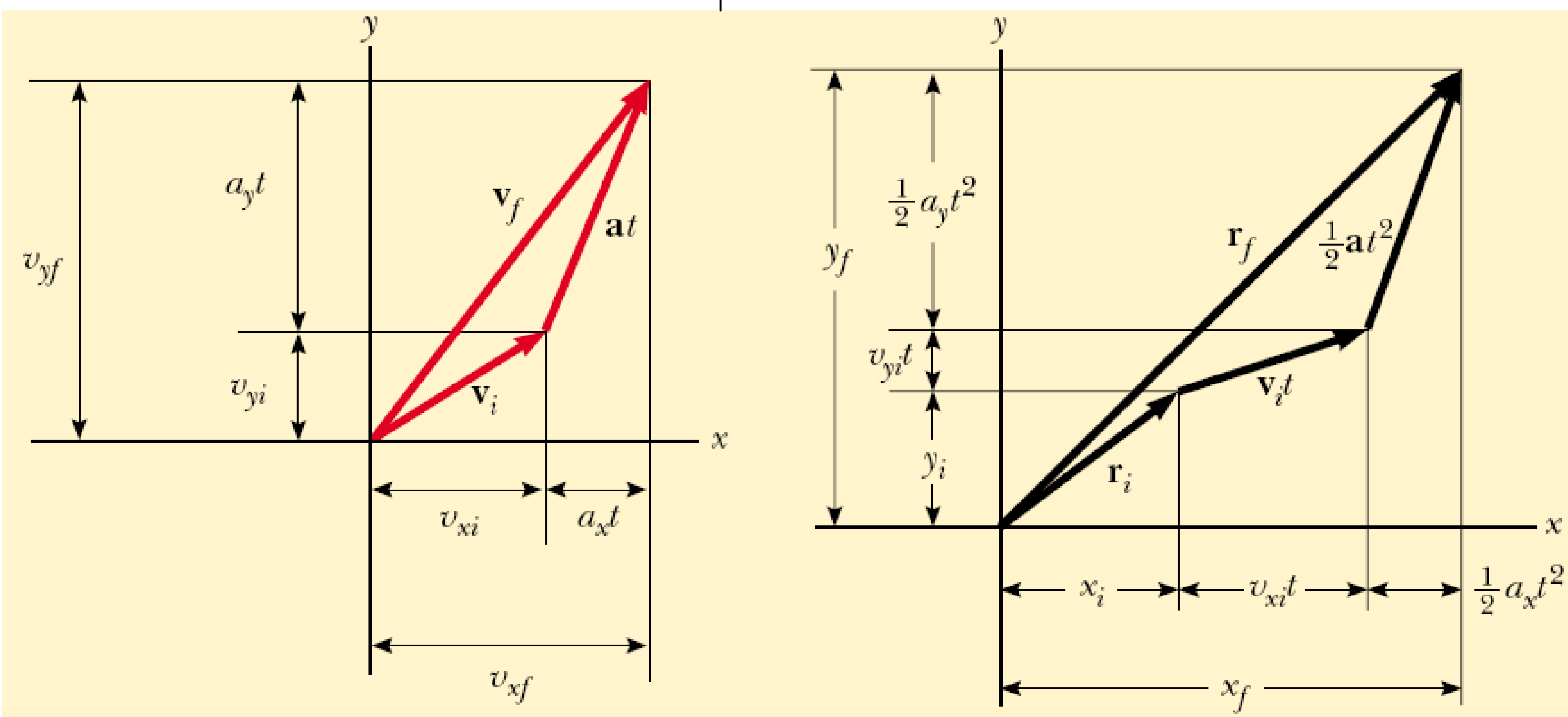
$$v_{xf} = v_{xi} + a_x t, \quad x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$v_{yf} = v_{yi} + a_y t, \quad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

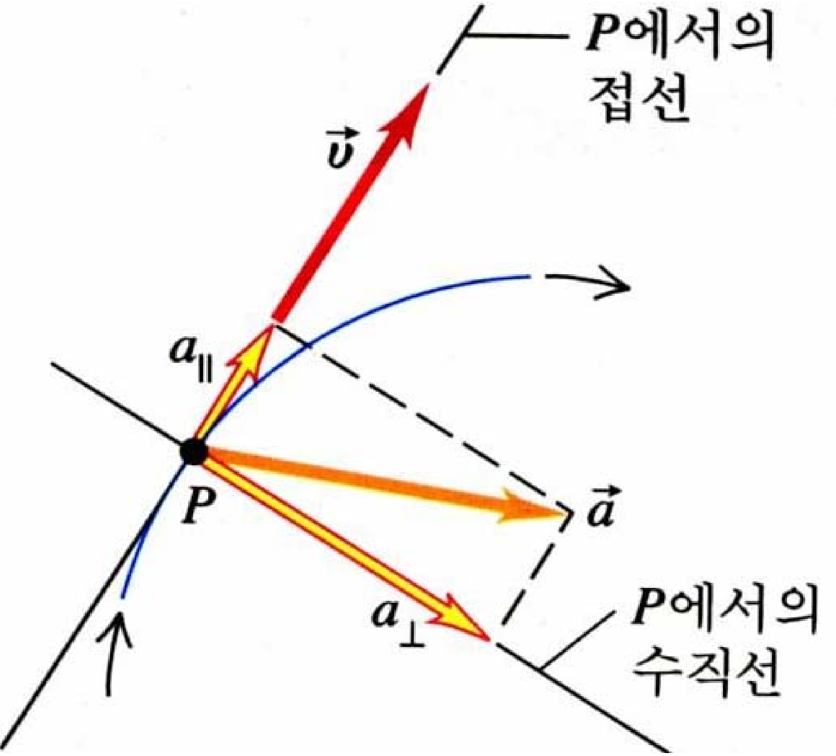
$$\vec{v}_f = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j} = \vec{v}_i + \vec{a}t$$

$$\vec{r}_f = \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\hat{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\hat{j}$$

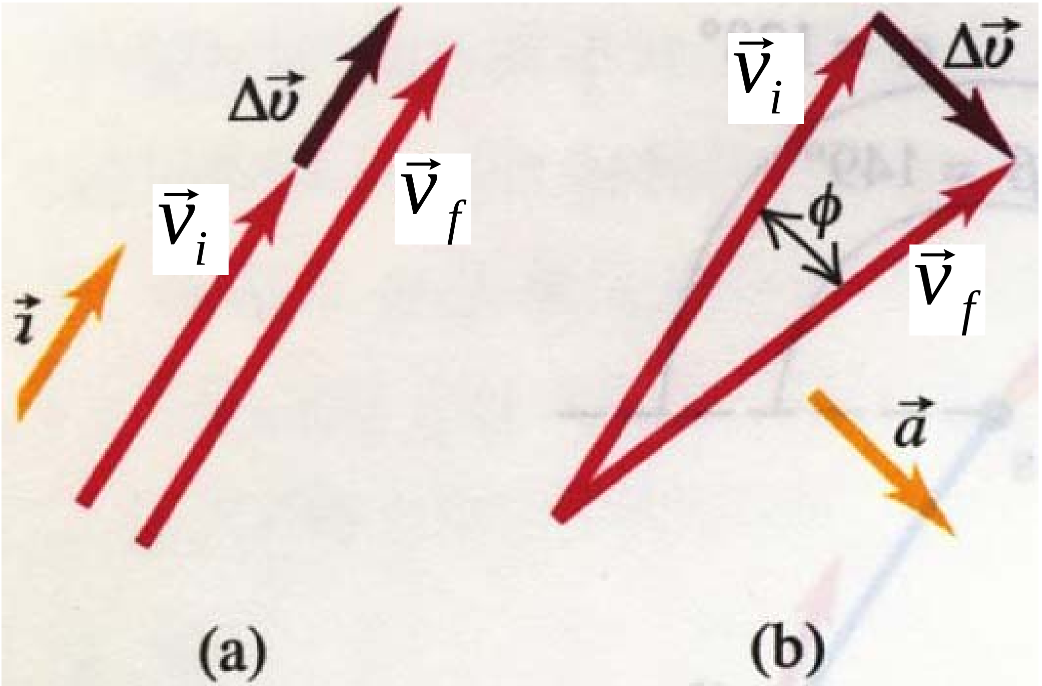
$$= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$



가속도의 방향 및 성분



비등속 원운동



(a)속력만 변하는 경우
(b)속력은 일정하고 방향만 변하는 경우 (등속 원운동)

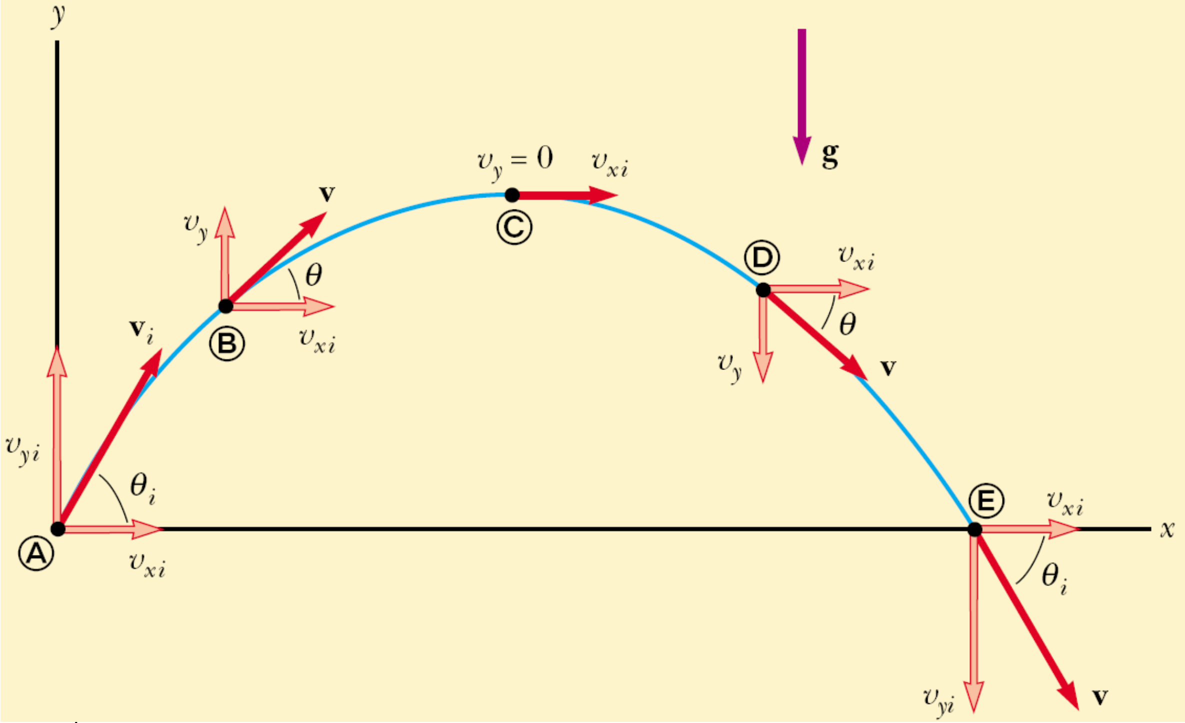
*가속도의 방향은 Δv 의 방향과 같다.

4-3 포물체 운동

가속도 $\vec{a} = a_x \hat{i} + a_y \hat{j}$

$= g(-\hat{j}) = -g\hat{j} \Leftarrow a_x = 0, a_y = -g$

초기속도 $\vec{v}_i = v_{xi}\hat{i} + v_{yi}\hat{j}$



$x -$ 성분 속도(v_x) 및 위치(x)

$v_x = v_{xi}$

$x = x_i + v_{xi}t$

$y -$ 성분 속도(v_y) 및 위치(y)

$v_y = v_{yi} - gt$

$y = v_{yi}t - \frac{1}{2}gt^2$

$t = 0;\ x_i = y_i = 0$ 이고,

$v_{xi} = v_i \cos\theta,\ v_{yi} = v_i \sin\theta$ 이므로,

$x = (v_i \cos\theta)t$ -----(1)

$y = (v_i \sin\theta)t - \frac{1}{2}gt^2$ -----(2)

$v_{xi} = v_i \cos\theta$

$v_{yi} = v_i \sin\theta - gt$

$\tan\theta = \frac{v_{yi}}{v_{xi}}$

$$x = (v_i \cos\theta)t \quad \text{-----------} (1)$$

$$y = (v_o \sin\theta)t - \frac{1}{2}gt^2 \quad \text{----} (2)$$

(1)식의 $t = \dfrac{x}{v_i \cos\theta}$ 를 (2)에 대입하면

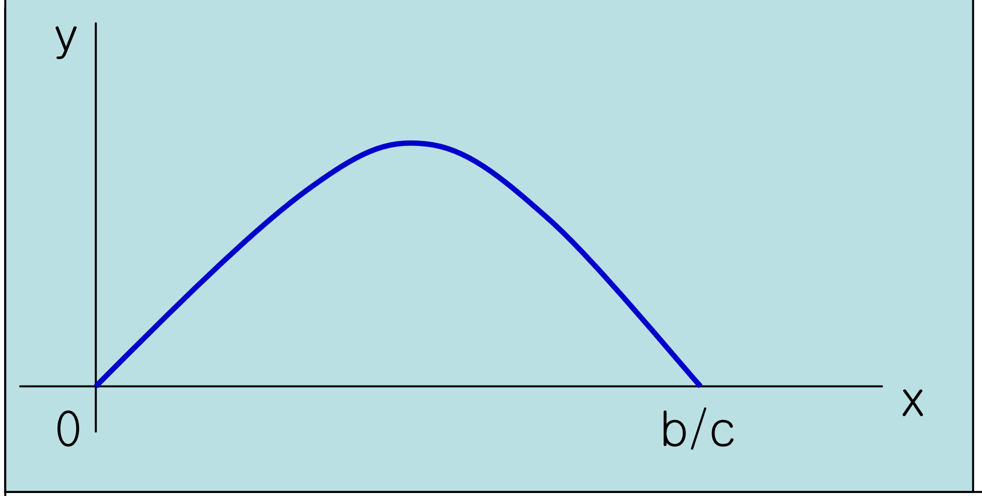
$$y = (v_i \sin\theta)\left(\frac{x}{v_i \cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{v_i \cos\theta}\right)^2$$

$$= (\tan\theta)x - \frac{g}{2(v_i \cos\theta)^2}x^2$$

$$b = \tan\theta, \; c = \frac{g}{2(v_i \cos\theta)^2}$$

$y = bx - cx^2 = x(b - cx)$ 인 포물선 함수.

$x = 0 \;\; or \;\; x = b/c$ 에서 $y = 0$



A) 사정거리(R)

$$R = \frac{b}{c} = \frac{\sin\theta}{\cos\theta}\frac{2(v_i \cos\theta)^2}{g}$$

$$= 2(v_i^2 / g)\sin\theta\cos\theta$$

B) 최대사거리($R_{\max}$)

$$R_{\max} = 2(v_i^2 / g)\sin\theta\cos\theta$$

$$= (v_i^2 / g)(2\sin\theta\cos\theta) = (v_i^2 / g)(\sin 2\theta)$$

최대값을 갖기 위해서

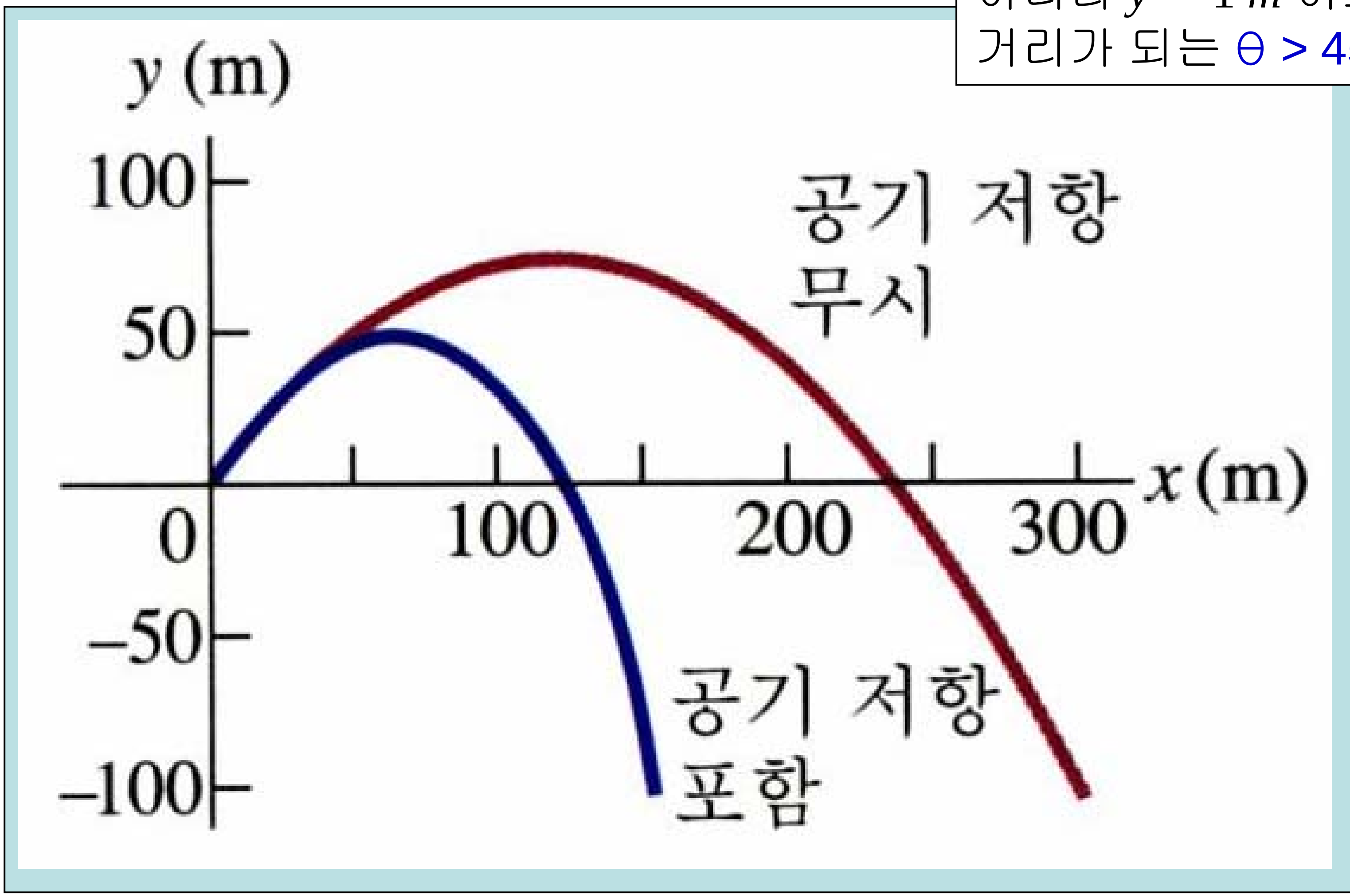
$$2\theta = 90^o, \;\; \theta = 45^o$$

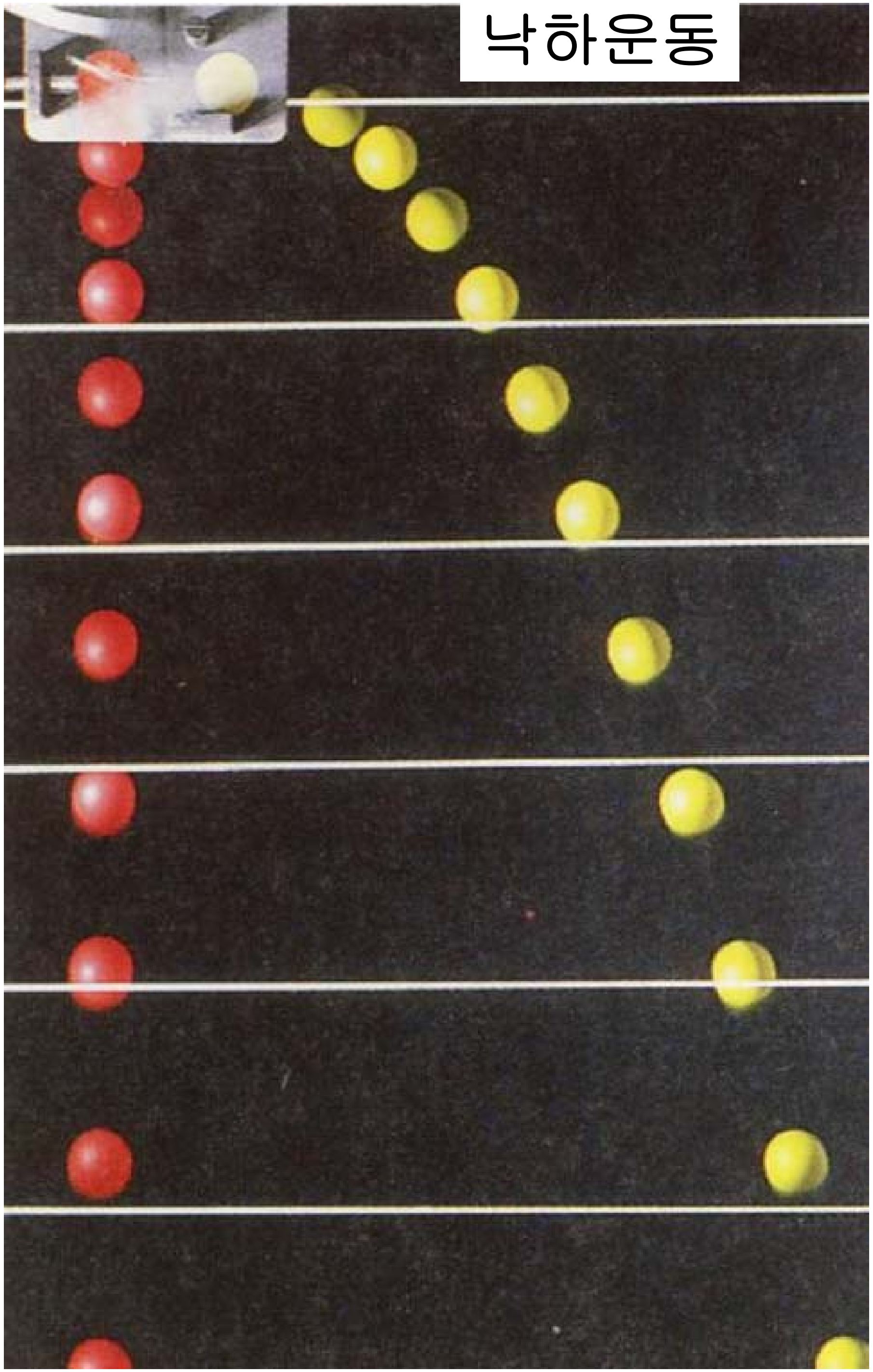
v_i = 50 m/s, Θ = 53.1°일 때,
홈런공의 포물선 운동

<실제 상황>

•공기의 저항을 고려해야 함.

•공이 출발하는 위치가 $y = 0$ 이 아니라 $y \approx 1\ m$ 이므로 최대사거리가 되는 Θ > 45° 이다.



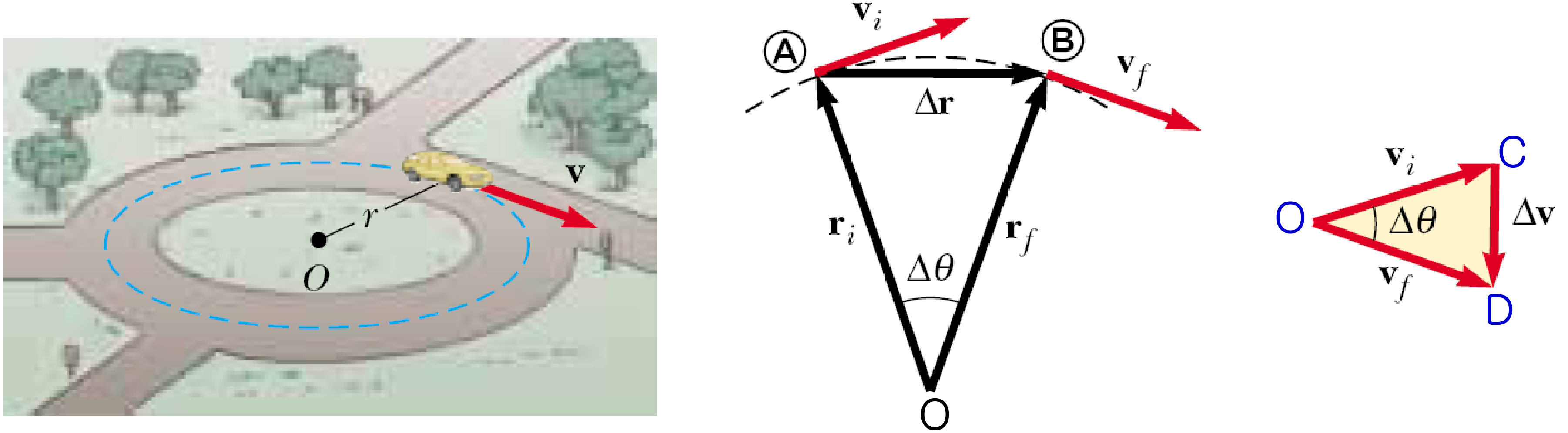


낙하운동

테니스공의 운동

화산분출

4-4. 등속 원운동



(a) (b) (c)

△OAB와 △OCD는 닮은꼴(SAS)

$$\frac{|\Delta\vec{v}|}{v_i} = \frac{|\Delta\vec{r}|}{r_i} \Rightarrow |\Delta\vec{v}| = \frac{v}{r}\Delta r$$

$$a = \lim_{\Delta t\to 0}\frac{|\Delta\vec{v}|}{\Delta t} = \lim_{\Delta t\to 0}\frac{v}{r}\frac{\Delta r}{\Delta t}$$

$$= \frac{v}{r}\lim_{\Delta t\to 0}\frac{\Delta r}{\Delta t} = \frac{v^2}{r} \quad (\because v = \lim_{\Delta t\to 0}\frac{\Delta r}{\Delta t})$$

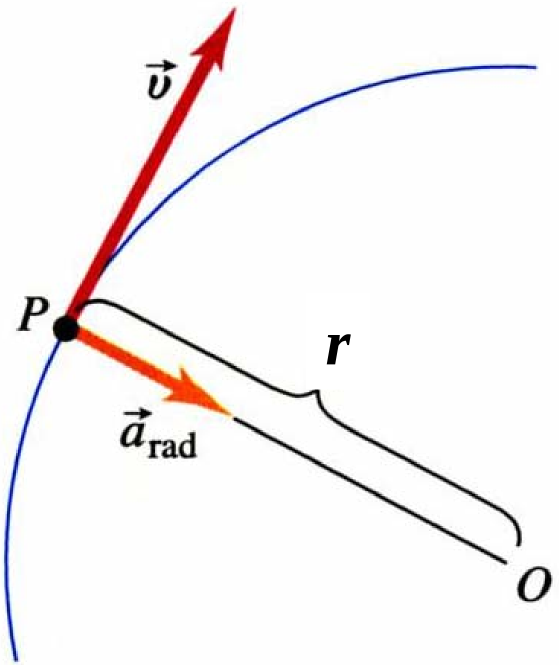
$a_c = \dfrac{v^2}{R}$, 구심가속도

또한, $v = \dfrac{2\pi r}{T}$ $(T =$ 주기$)$이므로

$$a_c = \frac{v^2}{r} = \left(\frac{2\pi r}{T}\right)^2\frac{1}{r} = \frac{4\pi^2 r}{T^2}$$

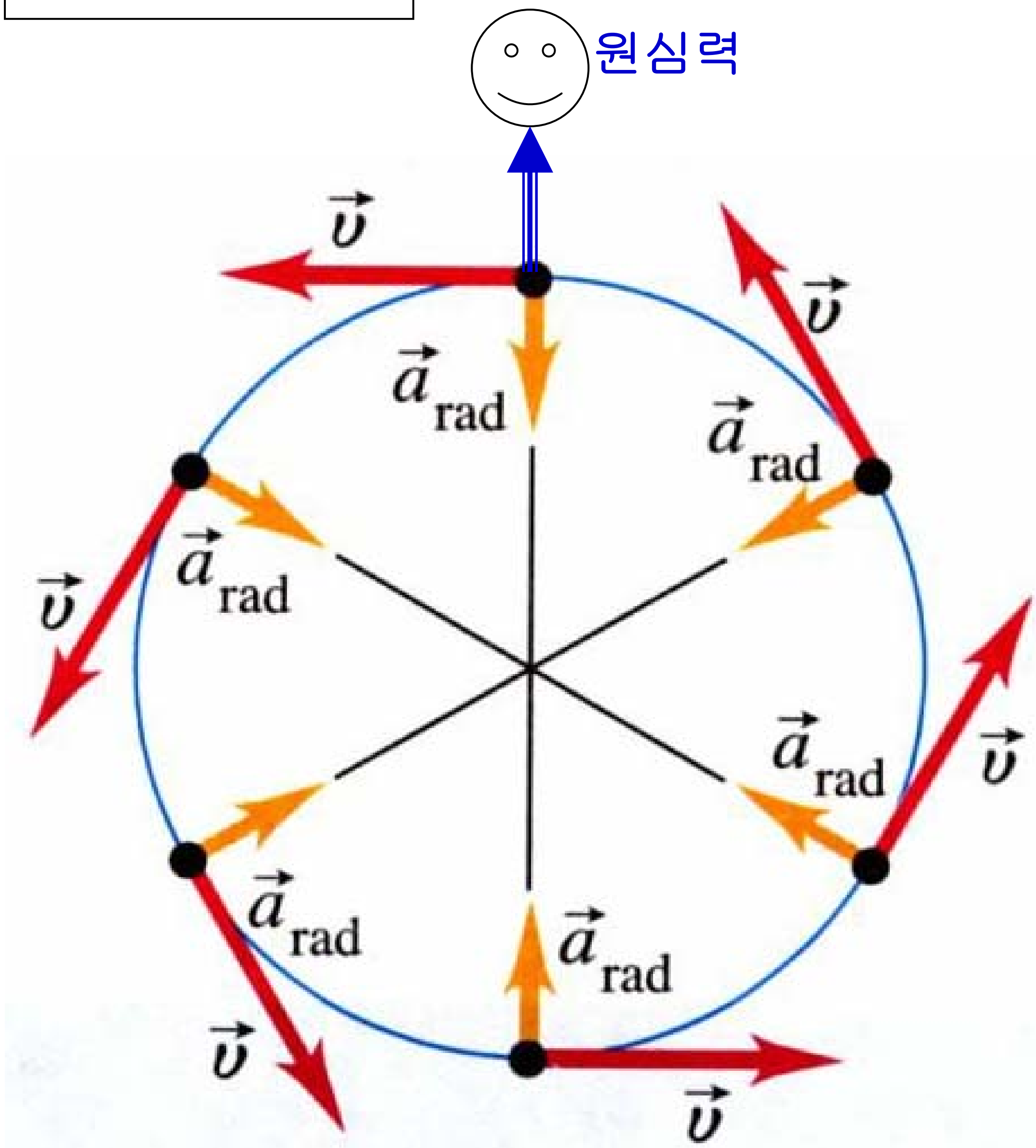
$$\vec{a}_c = \frac{v^2}{r}(-\hat{r}) = \frac{4\pi^2 r}{T^2}(-\hat{r})$$

($\hat{r}$: 반경방향 단위벡터)

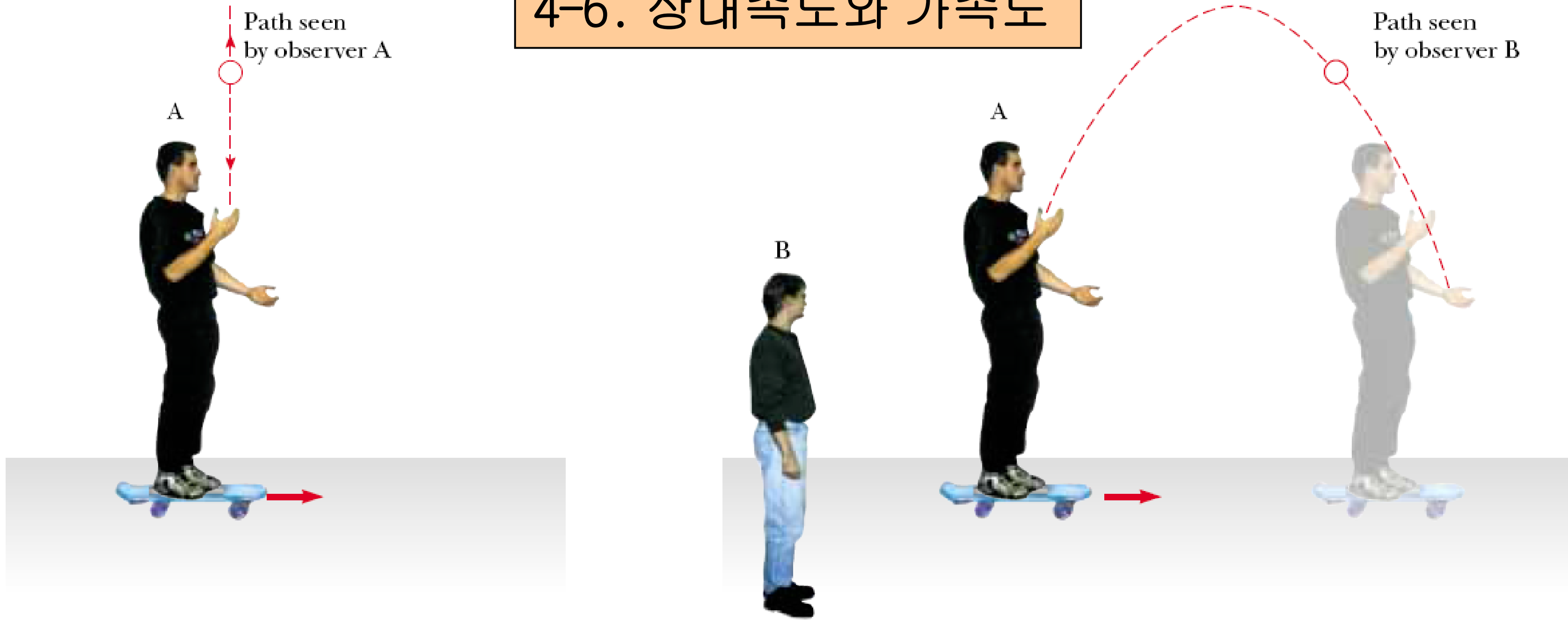


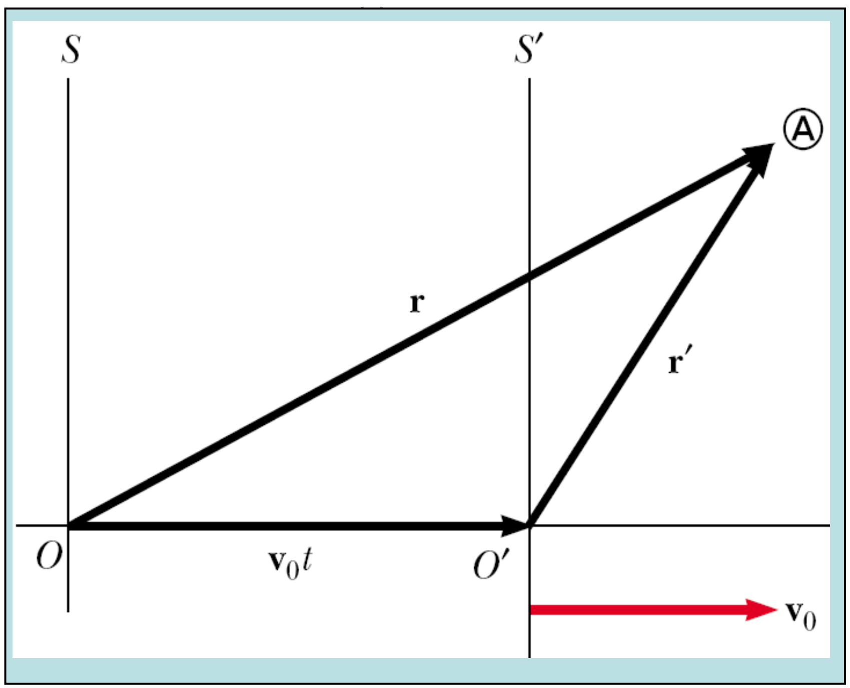
(c)

원심력과 구심력



4-6. 상대속도와 가속도





좌표변환: $\vec{r}' = \vec{r} - \vec{v}_0 t$

속도변환: $\dfrac{d\vec{r}'}{dt} = \dfrac{d\vec{r}}{dt} - \vec{v}_0$

$\vec{v}' = \vec{v} - \vec{v}_0$

가속도: $\dfrac{d\vec{v}'}{dt} = \dfrac{d\vec{v}}{dt} - \dfrac{d\vec{v}_0}{dt}$

등속운동($\vec{v}_0$ = 일정)인 경우

$\dfrac{d\vec{v}_0}{dt} = 0, \qquad \vec{a}' = \vec{a}$ (가속도 일정)

1차 숙제 (1~4장)

2장: 1, 3, 6, 10, 13
4장: 3, 4, 5, 6, 9